MATH 2260 Calculus II

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Name:

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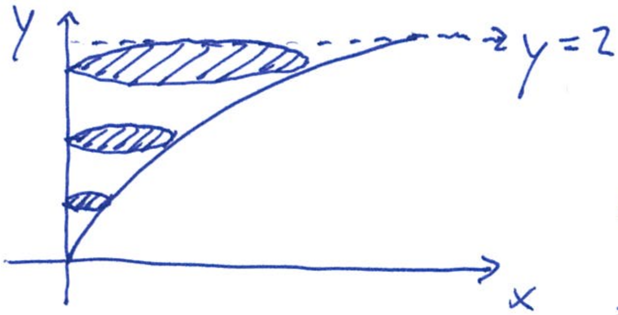
Practice Quiz No. 1

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 *Find the volume of the solid that lies between the planes perpendicular to the y-axis at $y = 0$ and $y = 2$. The cross-sections perpendicular to the y-axis are circular discs with diameters running from the y-axis to the parabola $x = \sqrt{3}y^2$.*



$$V = \int_0^2 A(y)\,dy$$

$$A(y) = \pi r^2 = \pi\left(\frac{\sqrt{3}y^2}{2}\right)^2$$

$$\Longrightarrow V = \int_0^2 \pi\left(\frac{\sqrt{3}y^2}{2}\right)^2 dy$$

$$= \frac{3\pi}{4}\int_0^2 y^4\,dy = \frac{3\pi}{4}\left[\frac{y^5}{5}\right]_0^2 = \frac{3\pi}{20}(2^5 - 0^5) = \frac{24\pi}{5}$$

Problem 2 *Integrate*

$$\int \frac{\cos(5x)}{e^{\sin(5x)}}dx$$

$u = \sin(5x)$

$du = \cos(5x)5\,dx$

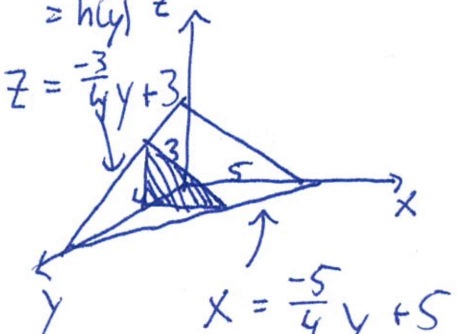
$$\longrightarrow \int \frac{\cos(5x)}{e^{\sin(5x)}}dx = \int \frac{\cos(5x)}{e^u}\left(\frac{5}{5}\right)dx$$

$$= \frac{1}{5}\int e^{-u}(\cos(5x)5\,dx) = \frac{1}{5}\int e^{-u}du = \frac{-1}{5}e^{-u} + C$$

$$= \frac{-1}{5}e^{-\sin(5x)} + C$$

Problem 3 *Find the volume of the tetrahedron shown below:*

$z = -\frac{3}{4}y + 3 = h(y)$



$x = -\frac{5}{4}y + 5 = b(y)$

~~Tetrahedron.png~~

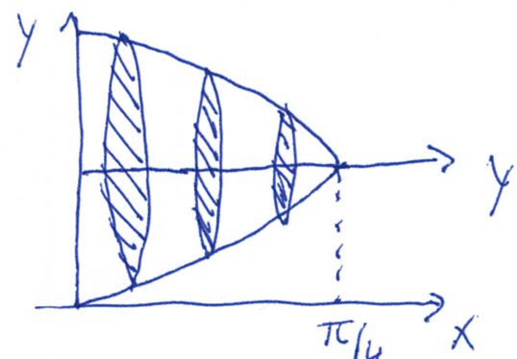
I'm choosing to integrate with respect to y.

$$V = \int_0^4 A(y)\,dy \quad , \quad A(y) = \frac{1}{2} b(y) h(y) = \frac{1}{2}\left(-\frac{5}{4}y + 5\right)\left(-\frac{3}{4}y + 3\right)$$

$$\Rightarrow V = \int_0^4 \frac{1}{2}\left(-\frac{5}{4}y+5\right)\left(-\frac{3}{4}y+3\right)dy = \frac{15}{2}\int_0^4 \left(-\frac{1}{4}y+1\right)\left(-\frac{1}{4}y+1\right)dy$$

$$= \frac{15}{2}\int_0^4 \frac{1}{16}y^2 - \frac{1}{2}y + 1\,dy = \frac{15}{2}\left(\frac{1}{16}\left[\frac{y^3}{3}\right]_0^4 - \frac{1}{2}\left[\frac{y^2}{2}\right]_0^4 + [y]_0^4\right) = 10$$

Problem 4 *Find the volume of the solid generated by revolving the region in the first quadrant bounded above by the line $y = \cancel{\sqrt{3}}$ $\sqrt{2}$ and below by the curve $y = \sec(x)\tan(x)$ about the line $y = \cancel{\sqrt{3}}$. $\sqrt{2}$*



Solve $\sqrt{2} = \sec(x)\tan(x)$ by guessing and checking $\Rightarrow x = \pi/4$.

$$V = \int_0^{\pi/4} A(x)\,dx \quad , \quad A(x) = \pi r^2 = \pi\left(\sqrt{2} - \sec(x)\tan(x)\right)^2$$

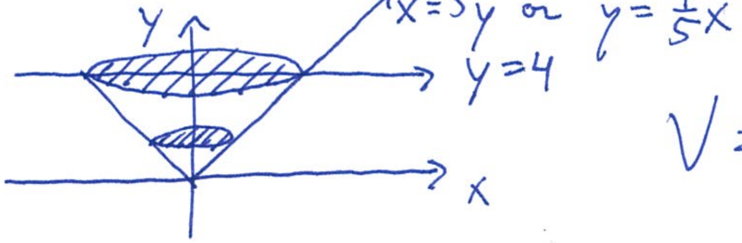
$$V = \int_0^{\pi/4} \pi\left(\sqrt{2} - \sec(x)\tan(x)\right)^2 dx = \pi\int_0^{\pi/4} 2 - 2\sqrt{2}\,\underbrace{\sec(x)\tan(x)} + \sec^2(x)\tan^2(x)$$

For $\int \sec^2(x)\tan^2(x)\,dx$, let $u = \tan(x)$, $du = \sec^2(x)dx$ $\quad \frac{d}{dx}\left[\sec(x)\right] = \sec(x)\tan(x)$

$$= \pi\left(2[x]_0^{\pi/4} - 2\sqrt{2}\left[\sec(x)\right]_0^{\pi/4} + \left[\frac{\tan^3(x)}{3}\right]_0^{\pi/4}\right)$$

$$= \pi\left(\frac{\pi}{2} - 4 + 2\sqrt{2} + \frac{1}{3}\right)$$

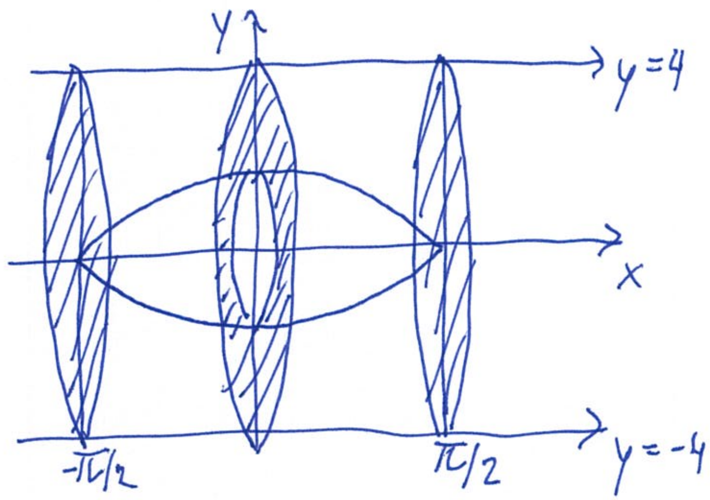
Problem 5 *Find the volume of the solid generated by revolving the triangular region in the first quadrant bounded by the line $y = 4$ and the line $x = 5y$ about the y-axis.*



$$V = \int_0^4 A(y)\,dy = \int_0^4 \pi(5y)^2\,dy$$
$$= 25\pi \int_0^4 y^2\,dy = 25\pi\left[\frac{y^3}{3}\right]_0^4$$
$$= \frac{25\pi}{3}(4^3 - 0^3) = \frac{1600\pi}{3}$$

Check using the volume of a cone, $V = \frac{1}{3}A_{base}\cdot h$
$$= \frac{1}{3}(20)^2\pi\, 4 = \frac{1600\pi}{3}$$

Problem 6 *Find the volume of the solid generated by revolving the region bounded above by the line $y = 4$, below by the curve $y = \sqrt{\cos(x)}$, $-\pi/2 \le x \le \pi/2$, rotated about the x-axis.*



$$V = \int_{-\pi/2}^{\pi/2} \pi\left(4^2 - \left(\sqrt{\cos(x)}\right)^2\right) dx$$
$$= \pi 16\left[x\right]_{-\pi/2}^{\pi/2} - \pi\left[\sin(x)\right]_{-\pi/2}^{\pi/2}$$
$$= \pi 16\left(\frac{\pi}{2} - \left(\frac{-\pi}{2}\right)\right)$$
$$- \pi(1 - (-1))$$
$$= \pi(16\pi - 2\pi) = 14\pi^2$$