MATH 2250 Calculus
Eric Perkerson

Name: Solutions
Date: April 25, 2017

Quiz No. 9

Problem 1 State the following theorems:

a Fundamental Theorem of Calculus, part I.

See notes

b Fundamental Theorem of Calculus, part II.

See notes

Problem 2 Find the values of the following definite integrals:

a $\int_0^2 x(x-3)dx$ $= \int_0^2 x^2 - 3x\, dx = \left[\frac{x^3}{3} - 3\frac{x^2}{2} \right]_0^2$

$= \frac{2^3}{3} - \frac{3}{2}2^2 - [0] = \frac{8}{3} - 6 = \frac{8-18}{3} = \frac{-10}{3}$

b $\int_1^{32} x^{-6/5}dx$ $= \left[\frac{x^{-6/5+1}}{-6/5+1} \right]_1^{32} = \left[\frac{x^{-1/5}}{-1/5} \right]_1^{32} = -5\left[32^{-1/5} - 1^{-1/5} \right]$

$= -5\left[\frac{1}{\sqrt[5]{32}} - 1 \right] = -5\left[\frac{1}{2} - 1 \right] = \frac{5}{2}$

c $\int_0^{\pi/3} 2\sec^2(x)dx)$ $= 2\int_0^{\pi/3} \sec^2(x)\, dx = 2\left[\tan(x) \right]_0^{\pi/3} = 2\left(\tan\left(\frac{\pi}{3}\right) - 0 \right)$

$= 2\tan\left(\frac{\pi}{3}\right)$

$= 2\sqrt{3}$

d $\int_0^{\pi/4} \tan^2(x)dx$ (Hint: You should be using a trig identity.) Use: $1 + \tan^2(x) = \sec^2(x)$

$= \int_0^{\pi/4} \sec^2(x) - 1\, dx = \left[\tan(x) - x \right]_0^{\pi/4} = \tan\left(\frac{\pi}{4}\right) - \frac{\pi}{4} - [0]$

$= \tan\left(\frac{\pi}{4}\right) - \frac{\pi}{4}$

$= 1 - \frac{\pi}{4}$

Problem 3 Find the derivatives of the following functions:

a $F(x) = \int_0^{\sqrt{x}} \cos(t)dt$

$$F'(x) = \cos(\sqrt{x})\left(\frac{1}{2}x^{-1/2}\right)$$

b $F(x) = \int_1^{\sin(x)} 3t^2 dt$

$$F'(x) = 3\sin^2(x)\cos(x)$$

Problem 4 Find the following antiderivatives:

a $\int 2\sqrt{2x+1}dx$ $= \int \sqrt{u}\, du = \frac{u^{3/2}}{3/2} + C = \frac{(2x+1)^{3/2}}{3/2} + C$

$u = 2x+1 \quad du = 2dx$

b $\int x^2 e^{(x^3)}dx$ $= \frac{1}{3}\int 3x^2 e^{x^3} dx = \frac{1}{3}\int e^u du = \frac{1}{3}e^u + C$

$u = x^3 \quad du = 3x^2 dx$

$= \frac{1}{3} e^{x^3} + C$

c $\int \cos^2(x)dx$ (Hint: Remember that $\cos^2(x) = \frac{1+\cos(2x)}{2}$)

$= \int \frac{1+\cos(2x)}{2} dx = \frac{1}{2}\int 1 + \cos(2x) dx = \frac{1}{2}\left(x + \int \cos(2x) dx\right)$

$= \frac{1}{2}\left(x + \frac{1}{2}\int \cos(2x) 2 dx\right) = \frac{1}{2}\left(x + \frac{1}{2}\int \cos(u) du\right) = \frac{1}{2}\left(x + \frac{1}{2}\sin(u)\right) +$

$u = 2x \quad du = 2dx$

$= \frac{1}{2}\left(x + \frac{1}{2}\sin(2x)\right) +$

d $\int \frac{1}{x\log(x)}dx$

$u = \log(x) \quad du = \frac{1}{x} dx$

$\Rightarrow \int \frac{1}{x\log(x)} dx = \int \frac{1}{u} du = \log(|u|) + C$

$= \log(|\log(x)|) + C$