MATH 2250 Calculus I
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Name: Answers
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Quiz No. 6

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 Given the function $f(x) = x^3 - 3x + 4$

a *Find all critical points of* $f(x)$.

$f'(x) = 3x^2 - 3$, polynomial, so always exists

$f'(x) = 0 = 3x^2 - 3 \Rightarrow 3 = 3x^2 \Rightarrow 1 = x^2 \Rightarrow x = \pm 1$

are the only critical points.

b *Find the minimum and maximum of* $f(x)$ *on the interval* $[-3/2, 3/2]$.

x	$f(x)$
$-3/2$	$\frac{-27}{8} + \frac{9}{2} + 4 = \frac{-27}{8} + \frac{36}{8} + \frac{32}{8} = \frac{41}{8} \approx 5$
-1	$-1 + 3 + 4 = 6 = \max_{x \in [-3/2, 3/2]} (f(x))$
1	$1 - 3 + 4 = 2 = \min_{x \in [-3/2, 3/2]} (f(x))$
$3/2$	$\frac{27}{8} - \frac{9}{2} + 4 = \frac{27}{8} - \frac{36}{8} + \frac{32}{8} = \frac{23}{8} \approx 3$

Problem 2 Given the function $f(x) = x^{1/3}$

a *Find all critical points of* $f(x)$.

$f'(x) = \frac{1}{3}x^{-2/3}$

$= \frac{1}{3x^{2/3}}$

$f'(x) = 0$ has no solutions.

$f'(x)$ DNE when $x = 0 \Rightarrow x = 0$ is only critical point ✓

b *Find the minimum and maximum of* $f(x)$ *on the interval* $[-8, 8]$

x	$f(x)$
-8	$(-8)^{1/3} = -2$
0	$(0)^{1/3} = 0$
8	$(8)^{1/3} = 2$

Problem 3 Given the function $f(x) = \sqrt{1+x^2}$

a *Find all critical points of* $f(x)$.

$$f'(x) = \frac{1}{2}(1+x^2)^{-1/2}(2x) = \frac{x}{\sqrt{1+x^2}}$$

Always exists because $1+x^2 \geq 1 > 0 \Rightarrow \sqrt{1+x^2} > 0$

$\Rightarrow f'(x) = 0 \Rightarrow x = 0$ is only critical point.

b *Find the minimum and maximum of* $f(x)$ *on the interval* $[-1, 1]$

x	$f(x)$	
-1	$\sqrt{2}$	$= \max_{x \in [-1,1]} (f(x))$
0	1	$= \min_{x \in [-1,1]} (f(x))$
1	$\sqrt{2}$	