MATH 2250 Calculus
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Name: Solutions
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Quiz No. 5

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 (25 points) State both parts of the Fundamental Theorem of Calculus.

a Part I:

If f is continuous on $[a,b]$, then

$F(x) = \int_a^x f(t)\,dt$ has derivative

$F'(x) = f(x)$.

b Part II:

If f is continuous on $[a,b]$, and if F is an antiderivative of f, then

$\int_a^b f(x)\,dx = F(b) - F(a)$

Problem 2 (25 points) Find the value of the definite integral

$$\int_0^4 x^2 + 1dx$$

using the **DEFINITION** of the definite integral. You might need some of the following formulas:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Subintervals: $[0, \frac{4}{n}], [\frac{4}{n}, 2(\frac{4}{2})], \dots, [(i-1)(\frac{4}{n}), i(\frac{4}{n})], \dots$

Because f is increasing on $[0,4]$, U_n will use right end point, L_n will use left end points:

$$U_n = \sum_{i=1}^{n}\left(\left(\frac{4i}{n}\right)^2 + 1\right)\left(\frac{4}{n}\right) = \frac{4}{n}\left[\sum_{i=1}^{n}\frac{16i^2}{n^2} + \sum_{i=1}^{n} 1\right] = \frac{4}{n}\left[\frac{16}{n^2}\sum_{i=1}^{n} i^2 + n\right]$$

$$= \frac{4^3}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) + 4 = \frac{4^3}{6}\left(\frac{(n^2+n)(2n+1)}{n^3}\right) + 4 = \frac{4^3}{6}\left(\frac{2n^3+3n^2+n}{n^3}\right) + 4$$

$$\Rightarrow \lim_{n\to\infty} U_n = 4 + \frac{4^3}{6}\lim_{n\to\infty}\left(\frac{2n^3+3n^2+n}{n^3}\right) = 4 + \frac{4^3}{6}(2)$$

$$L_n = \sum_{i=1}^{n}\left(\frac{(i-1)^2 4^2}{n^2} + 1\right)\left(\frac{4}{n}\right) = \frac{4}{n}\left[\sum_{i=1}^{n}\frac{(i^2-2i+1)4^2}{n^2} + \sum_{i=1}^{n} 1\right]$$

$$= \frac{4}{n}\left[\frac{4^2}{n^2}\left(\sum_{i=1}^{n}(i^2) + \sum_{i=1}^{n}(-2i) + \sum_{i=1}^{n} 1\right) + n\right] = \frac{4}{n}\left[\frac{4^2}{n^2}\left(\frac{n(n+1)(2n+1)}{6} - 2\left(\frac{n(n+1)}{2}\right) + n\right) + n\right]$$

$$= \frac{4}{n}\left[\frac{4^2}{6}\left(\frac{2n^3+3n^2+n}{n^2}\right) - 4^2 2\left(\frac{n^2+n}{2}\right) + \frac{4^2}{n}\right] + n\Bigg] = \frac{4^3}{6}\left(\frac{2n^3+3n^2+n}{n^3}\right) - \frac{2\cdot 4^3}{2}\left(\frac{n^2+n}{n^3}\right) + \frac{4}{n^2} +$$

$$\Rightarrow \lim_{n\to\infty} L_n = \frac{4^3}{6}(2) + 4 \quad \Rightarrow \int_0^4 x^2 + 1\,dx = 4 + 2\left(\frac{4^3}{6}\right)$$

Problem 3 (25 points) Find the following definite integrals using the Fundamental Theorem of Calculus, part II.

a $\int_0^1 101x^{100} + 2dx$

$$= \left[x^{101} + 2x\right]_0^1 = 3$$

b $\int_0^{\pi/2} \cos(x)dx$

$$= \left[\sin(x)\right]_0^{\pi/2} = 1 - 0 = 1$$

c $\int_1^3 1/x + e^x dx$

$$= \left[\log(|x|) + e^x\right]_1^3 = \left[\log(3) + e^3\right] - \left[\log(1) + e^1\right]$$

$$= \log(3) + e^3 - e$$

d $\int_0^1 \frac{1}{1+x^2}dx$

$$= \left[\tan^{-1}(x)\right]_0^1 = \tan^{-1}(1) - \tan^{-1}(0) = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

e $\int_{-1}^2 3x + 5dx$

$$= \left[3\frac{x^2}{2} + 5x\right]_{-1}^2 = \left[3\frac{2^2}{2} + 10\right] - \left[\frac{3}{2} - 5\right]$$

$$= 6 + 10 - \frac{3}{2} + 5$$

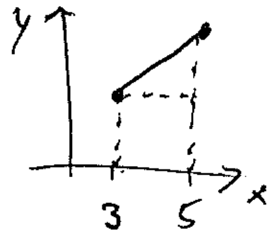
$$= 21 - \frac{3}{2}$$

$$= \frac{42}{2} - \frac{3}{2}$$

$$= \frac{39}{2}$$

Problem 4 (10 points) Draw a sketch of the graph of each function f(x) over the given interval.

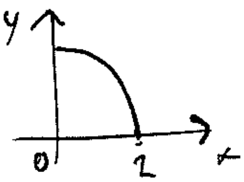
a $f(x) = 2x + 1$ on $[3, 5]$.



$\text{Area} = (5-3)(f(3)) + \frac{1}{2}(5-3)(f(5) - f(3))$

$= 2(7) + \frac{1}{2}(2)(11-7) = 18$

b $f(x) = \sqrt{4 - x^2}$ on $[0, 2]$.



$\text{Area} = \frac{1}{4}(\pi(2)^2) = \pi$

c $f(x) = 14$ on $[-2, 2]$.

$\text{Area} = (2-(-2))(14) = 40+16 = 56$

Problem 5 (15 points) Find the value of the definite integrals below using **AREA FORMULAS** of basic shapes.

a $\int_3^5 2x + 1dx$ $= 18$

b $\int_0^2 \sqrt{4 - x^2}dx$ $= \pi$

c $\int_{-2}^2 14dx$ $= 56$