

Quiz No. 1

Show all of your work, label your answers clearly, and do not use a calculator.

**Problem 1** (25 points) Evaluate the following limit:

$$\lim_{x \rightarrow 2} (x^2 + 5 + e^x)$$

(Be careful that everything you write down is true! Don't drop the lim symbol if the statement isn't true without it.)

$x^2 + 5 + e^x$  is continuous, hence

$$\lim_{x \rightarrow 2} (x^2 + 5 + e^x) = (2)^2 + 5 + e^{(2)} = 9 + e^2$$

**Problem 2** (25 points) Evaluate the following limit:

$$\lim_{x \rightarrow \infty} \left( \frac{8x^2 + 4x + 3}{x^2 + 10} \right)^{1/3}$$

$f(x) = x^{1/3}$  is continuous, hence

$$\lim_{x \rightarrow \infty} \left( \frac{8x^2 + 4x + 3}{x^2 + 10} \right)^{1/3} = \left( \lim_{x \rightarrow \infty} \frac{8x^2 + 4x + 3}{x^2 + 10} \right)^{1/3} = \left( \lim_{x \rightarrow \infty} \frac{8x^2 + 4x + 3 / \frac{1}{x^2}}{x^2 + 10 / \frac{1}{x^2}} \right)^{1/3}$$

$$= \left( \lim_{x \rightarrow \infty} \frac{8 + \frac{4}{x} + \frac{3}{x^2}}{1 + \frac{10}{x^2}} \right)^{1/3} = 8^{1/3} = 2$$

**Problem 3** (25 points) Evaluate the following limit

$$\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x} \left( \frac{\sqrt{2+x} + \sqrt{2}}{\sqrt{2+x} + \sqrt{2}} \right) = \lim_{x \rightarrow 0} \frac{2+x - 2}{x(\sqrt{2+x} + \sqrt{2})}$$

$$= \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{2+x} + \sqrt{2})} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{2+x} + \sqrt{2}} = \frac{1}{\sqrt{2} + \sqrt{2}} = \frac{1}{2\sqrt{2}}$$

**Problem 4** (25 points) Evaluate the following limit

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{|x - 1|}$$

Break into left and right limits

$$\lim_{x \rightarrow 1^+} \frac{x^2 - 1}{|x - 1|} = \lim_{x \rightarrow 1^+} \frac{(x+1)(x-1)}{x-1} = \lim_{x \rightarrow 1^+} (x+1) = 2$$

$$\lim_{x \rightarrow 1^-} \frac{x^2 - 1}{|x - 1|} = \lim_{x \rightarrow 1^-} \frac{(x+1)(x-1)}{-(x-1)} = \lim_{x \rightarrow 1^-} -(x+1) = -2$$

$$\text{So } \lim_{x \rightarrow 1} \frac{x^2 - 1}{|x - 1|} = \text{DNE}$$