MATH 2250 Calculus I
Eric Perkerson

Name:
Date: November 2, 2015

Practice Test No. 3

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 State the following theorems:

a Rolle's Theorem

b The Mean Value Theorem

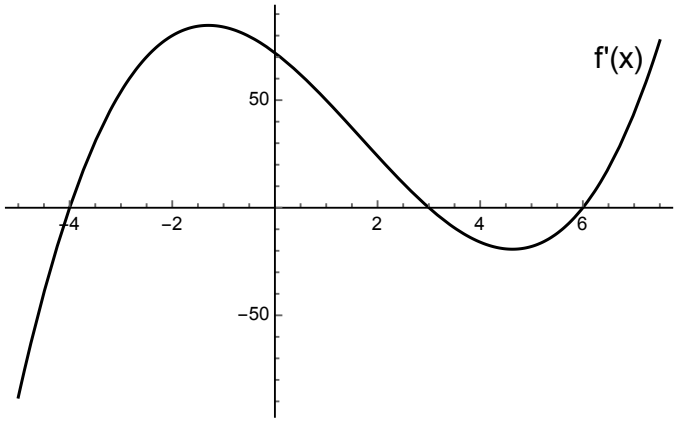
Problem 2

a Does the Mean Value Theorem apply to the function $f(x) = |x|$ on the interval $[-1, 1]$?

b Does the Mean Value Theorem apply to the function $g(x) = 1/x$ on the interval $[0, 1]$?

Problem 3 A box with a square base and an open top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimizes the amount of material used.

Problem 4 Below is a graph of the *derivative* of $f(x)$. This is a graph of $f'(x)$; do *not* make the mistake of thinking this is a graph of $f(x)$. Use this graph to answer the following questions about $f(x)$, $f'(x)$, and $f''(x)$.



a Where does the function $f(x)$ have critical points?

b Where does the function $f'(x)$ have critical points?

c Where does the function $f''(x)$ have critical points?

d Where is the function $f(x)$ increasing?

e Where is the function $f'(x)$ increasing?

f Where is the function $f''(x)$ increasing?

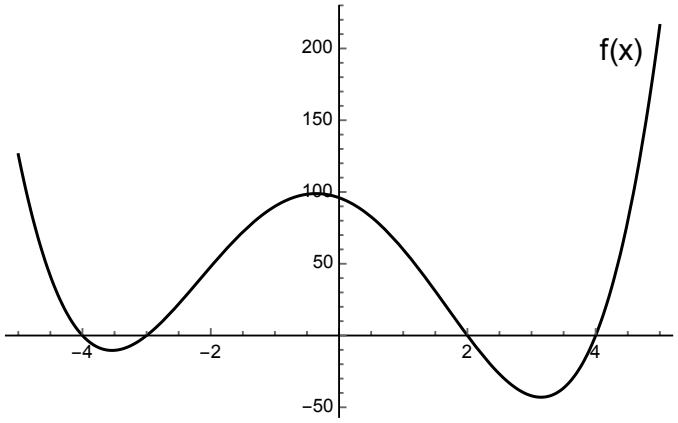
g Where does the function $f(x)$ have inflection points?

h Where does the function $f'(x)$ have inflection points?

i Where is the function $f(x)$ concave up?

j Where is the function $f'(x)$ concave up?

Problem 5 Below is a graph of $f(x)$. Use this graph to answer the following questions about $f(x)$, $f'(x)$, and $f''(x)$.



a Where does the function $f(x)$ have critical points?

b Where does the function $f'(x)$ have critical points?

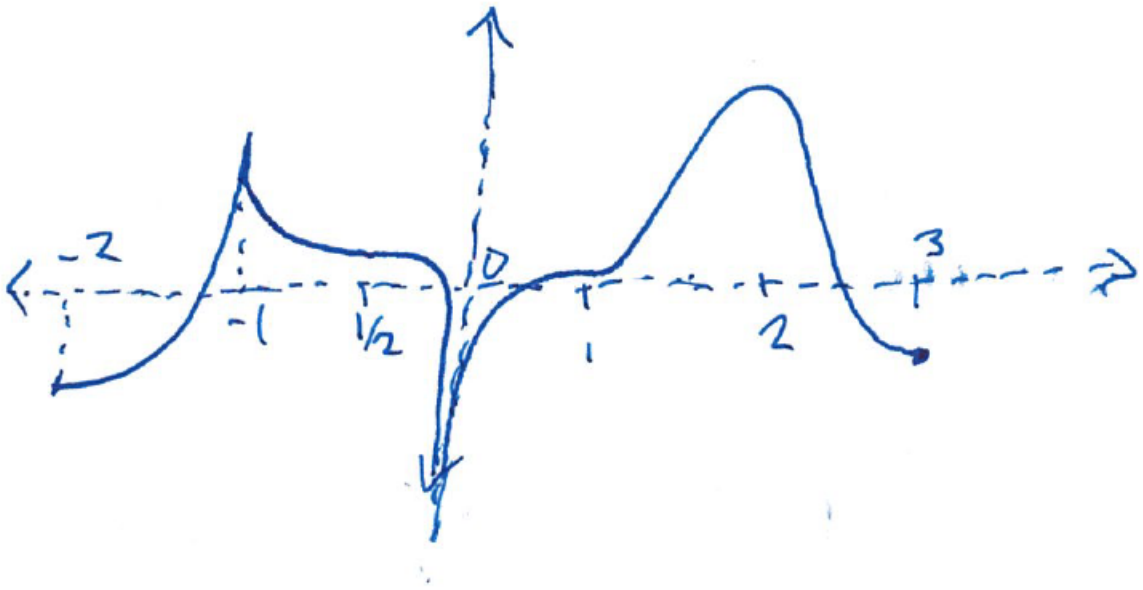
c Where is the function $f(x)$ increasing?

d Where is the function $f'(x)$ increasing?

e Where does the function $f(x)$ have inflection points?

f Where is the function $f(x)$ concave up?

Problem 6 For the given graph of $f(x)$, answer the questions below.



a Where does the function $f(x)$ have points where $f'(x)$ does not exist?

b Where does the function $f(x)$ have points where $f'(x) = 0$?

c Where does the function $f(x)$ have relative maxima?

d Where does the function $f(x)$ have relative minima?

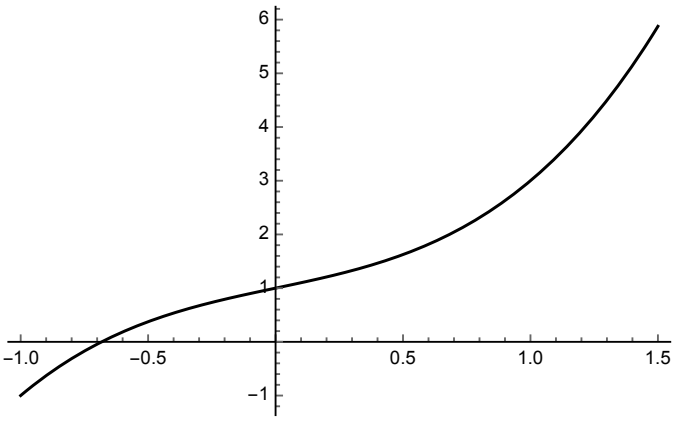
e Where does the function $f(x)$ have global maxima?

f Where does the function $f(x)$ have global minima?

Problem 7

a Use the function $f(x) = x^3 + x + 1$ and the starting point $x_0 = 1$ to run two iterations of Newton's method, i.e. find x_1 and x_2. (You do not have to simplify x_2).

b On the graph of $f(x)$ below, show graphically what Newton's method is doing. (You should be drawing some lines on the graph).



Problem 8 Evaluate the following limits, making sure that if you use L'Hôpital's rule you have written sufficient justification:

a $\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right)$

b $\lim_{x \to 0} \frac{3x^2}{\cos(x) - 1}$

c $\lim_{x \to 0^+} x^x$

Problem 9 Find the global maximum and global minimum of the function given by $f(x) = -(x+1)^{2/3} + 3$ on the interval $[-2, 2]$ and where each of these values occur.

Problem 10 Given the function $f(x) = (x+7)(x+4)(x+3)x$ on the interval $[-7, 0]$, answer the following questions:

a What are the x-intercepts of $f(x)$?

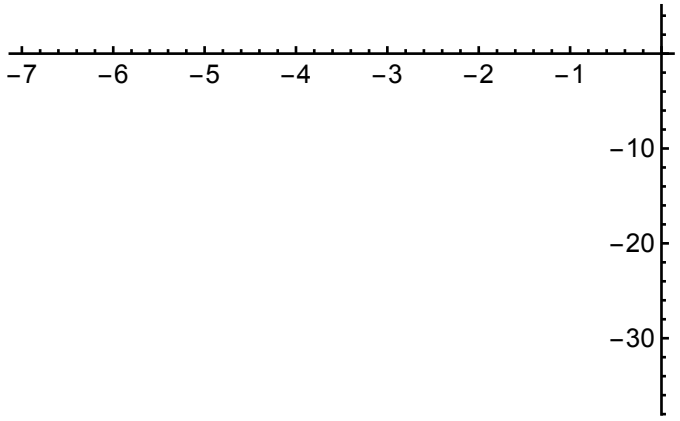
b I'll go ahead and tell you that the derivative is $f'(x) = 2(x+1)(x+6)(2x+7)$. What are the critical points of $f(x)$?

c On which intervals is $f(x)$ increasing?

d Classify the critical points of $f(x)$ as local maxima, local minima, or neither.

e What is/are the global maximum(s) and the global minimums of $f(x)$? (A function can have more than one global maximum or minimum if f attains the same value at those points.)

f Sketch the graph of $f(x)$. (Don't worry about concavity for this problem).



Problem 11 You are sitting in a classroom next to the wall looking at the blackboard at the front of the room. The blackboard is 12 ft long and starts 3 ft from the wall you are sitting next to. What is the maximum viewing angle, α?