MATH 2250 Calculus I
Eric Perkerson

Name:
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Practice Test No. 2

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 (15 points) Derive the derivatives of the given inverse trig functions *using reference triangles*:

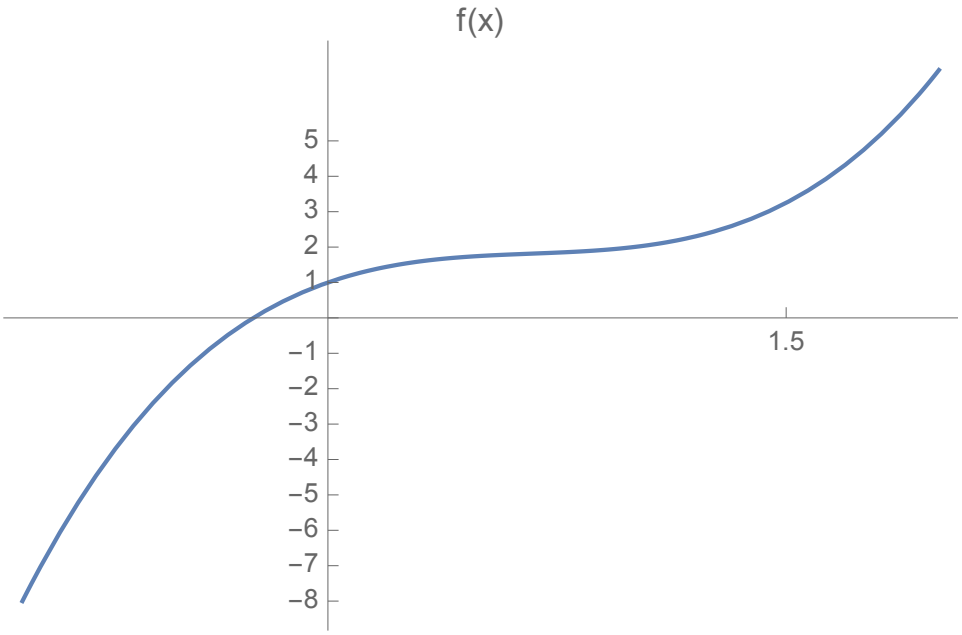
a $\dfrac{d}{dx}\arctan(x)$

a $\dfrac{d}{dx}\arcsin(x)$

Problem 2 (10 points) For full credit on this problem, make sure that you show your work by showing me how you are using the graph **and** by writing a sentence explaining what you are doing. (You should be drawing a line somewhere on the graph).

a Estimate $f(1.5)$

b Estimate $f'(1.5)$



Problem 3 (10 points) Find the following derivatives using derivative rules:

a $f(x) = 4x^3 + 2x^{-1}$, $f'(x) =$

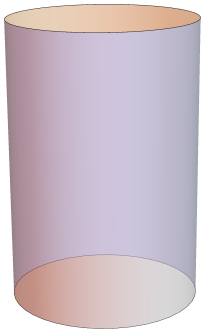
b $g(x) = e^x(5x^2 + x)$, $g'(x) =$

c $h(x) = 2\dfrac{\sin(x)}{\ln(x)}$, $h'(x) =$

d $y(x) = 2(3x + 50)^{2015}$, $y'(x) =$

e $u(x) = \dfrac{\ln(e^x) + \ln(\ln(x))}{5}$, $u'(x) =$

Problem 4 (20 points) A manufacturing plant receives cylindrical ingots of steel and then compresses them to make flattened steel sheets. The radius of the of the **cylindrical** ingots it receives is $10cm$ and the height of the **cylindrical** ingots is $25cm$. The ingots are heated, and then the press the plant uses to flatten the ingots crushes them so that their height changes at a constant $-2cm/min$. Assuming that the ingots stay **cylindrical** as they are being crushed, at what rate is the radius of the ingots changing when the height of the ingots is equal to $5cm$? (*Hint: the volume of the ingots as they are being crushed stays constant.*)



Problem 5 (15 points) Given the equation $y^4 = y^2 - x^2$:

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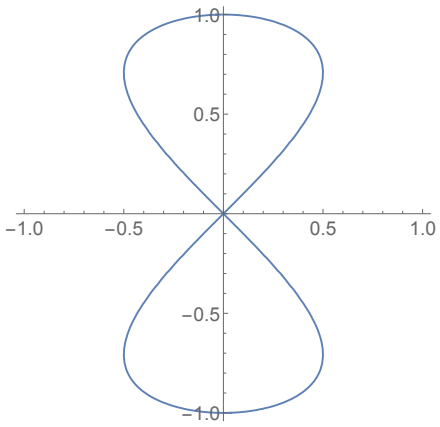
a Use implicit differentiation to show that $y'(x) = \dfrac{-x}{2y^3 - y}$

b Find the tangent line at the point $\left(\dfrac{\sqrt{3}}{4}, \dfrac{1}{2}\right)$

c Given the curve that satisfies the equation $y^4 = y^2 - x^2$, on the same graph, sketch the tangent line you just found.



Problem 6 (20 points) Astronomers estimate that on a certain exo-planet, a rock thrown upwards from $10m$ above the surface with an initial velocity of $12m/sec$ would have a height above the surface given by the equation $s(t) = -14t^2 + 12t + 10$

a Find the rock's velocity and acceleration as function of time t.

$v(t) =$ _____ $a(t) =$ _____

b How long does it take the rock to reach its highest point?

c How high does the rock go?

d How long does it take the rock to reach half its maximum height?

e How long is the rock aloft?

Problem 7 (10 points) Use the technique of linearization to estimate the value of $\sqrt[3]{9}$.