MATH 2250 Calculus I
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Practice Test No. 1

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 State the (informal) definition of a limit. (Careful, it's surprising easy to make a logical mistake when rephrasing this definition.)

Problem 2 Evaluate the limits below. You do not have to show any work for this problem.

a $\lim_{x \to -2} \frac{x+2}{x^2-4}$

b $\lim_{x \to 4^-} \frac{1}{\sqrt{4-x}}$

c $\lim_{x \to \infty} \frac{x^2-1}{8x^2-8x}$

d $\lim_{x \to 3} \frac{x-3}{x^2-x-6}$

e $\lim_{x \to 0} \frac{\sin(x)}{x}$

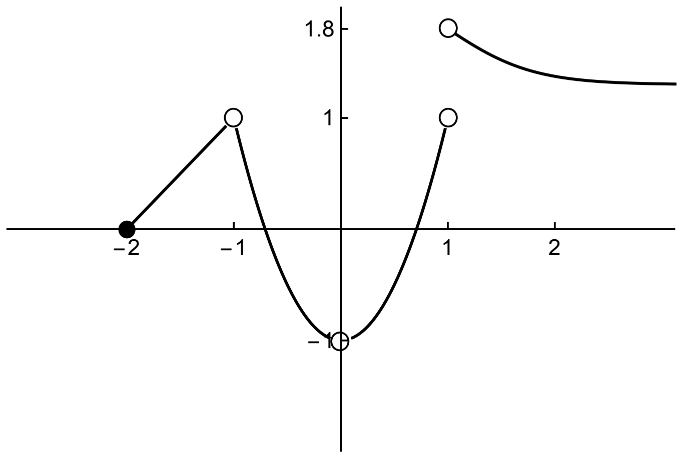
f $\lim_{x \to \infty} \frac{\sin(x)}{x}$

Problem 2 Find the average rate of change of the function $f(x) = 5x^2 + 4$ on the interval $[-1, 2]$.

Problem 3 Prove that the equation $x^3 - e^x = 0$ has a solution on the interval $[0, 3]$. Carefully justify your answer. (Hint: Remember that $2.718 \approx e < 3$).

Problem 4 State the definition of what it means for a function $f(x)$ to be continuous at a point a (assume that a is not an endpoint of the domain).

Problem 5 Use the below graph to answer this question:



a Find $\lim_{x \to -1} f(x)$.

b Find $\lim_{x \to 1^+} f(x)$.

c Find $\lim_{x \to 1^-} f(x)$.

d Find $\lim_{x \to 1} f(x)$.

e Is $f(x)$ continuous at $x = -1$? Why or why not?

f Is $f(x)$ continuous at $x = -2$? Why or why not?

g Is $f(x)$ continuous? Why or why not?

Problem 4 Evaluate, showing your work:

$$\lim_{x\to 0} 5x^2 + 10x + \sin(x) + e^x$$

Problem 5 Evaluate, showing your work:

$$\lim_{x\to 1} \frac{-x^2 - 4x}{x^2 + 5x - 4}$$

Problem 6 Evaluate, showing your work:

$$\lim_{x\to 4} \frac{x - 4}{x^2 - 3x - 4}$$

Problem 7

a Find the slope of the tangent line (i.e. the instantaneous rate of change) for the function $f(x) = \dfrac{1}{3x+5}$ at the point $x = 1$, i.e. find

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

b Now find the equation of the tangent line for the function $f(x) = \dfrac{1}{3x+5}$ at the point $x = 1$

Problem 8

a Find the slope of the tangent line (i.e. the instantaneous rate of change) for the function $f(x) = \sqrt{2x+1}$ at the point $x = 3$, i.e. find

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

b Now find the equation of the tangent line for the function $f(x) = \sqrt{2x+1}$ at the point $x = 3$

Problem 9 Find the value of a such that the function $f(x)$ given below is continuous on the domain $(-\infty, \infty)$. Once you have found the value of a, explain why $f(x)$ is continuous, citing the definition of continuity.

$$f(x) = \begin{cases} a\sin(x), & x \leq \pi/2 \\ ax^2 - 3, & x > \pi/2 \end{cases}$$

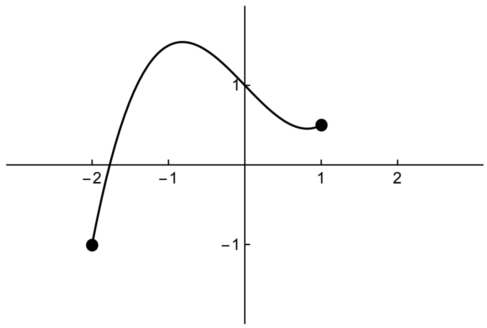
Problem 10 Find all vertical, horizontal, and slant asymptotes of the following functions:

a $f(x) = \dfrac{3x^2 + 4x + 1}{x^2 - 1}$

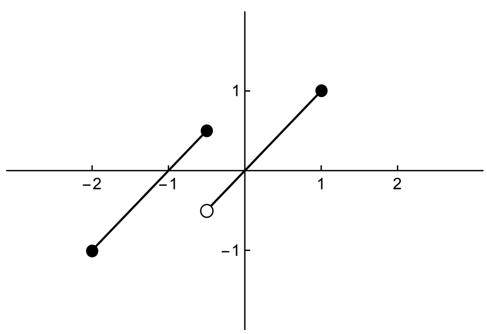
b $f(x) = \dfrac{x^2 - 3x + 2}{3x - 4}$

Problem 11 A function $f(x)$ is said to have the *intermediate value property* on an interval $[a, b]$ if, for any value $y = y_0$ between $f(a)$ and $f(b)$, there exists a value $x = c$ such that $f(c) = y_0$. With this definition, we can restate the Intermediate Value Theorem as follows: If $f(x)$ is continuous on $[a, b]$, then $f(x)$ has the intermediate value property on $[a, b]$. Use this to answer the following questions.

a Does the function pictured in the graph below have the intermediate value property on the interval $[-2, 1]$?



b Does the function pictured in the graph below have the intermediate value property on the interval $[-2, 1]$?



c In general, if a function $f(x)$ has the intermediate value property on the interval $[a, b]$, how does the range of the function $f(x)$ relate to the interval $[f(a), f(b)]$?