MATH 2250 Calculus I
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Name: Answers
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Practice Quiz No. 9

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 Find the generalized antiderivative of $f(x) = -2x^5 + 3x + 2$

$$\int f(x)\,\partial x = \int -2x^5 + 3x + 2\,\partial x$$
$$= \int -2x^5\,\partial x + \int 3x\,\partial x + \int 2\,\partial x$$
$$= -2\int x^5\,\partial x + 3\int x\,\partial x + 2\int 1\,\partial x$$
$$= -2\left(\frac{x^6}{6}\right) + 3\left(\frac{x^2}{2}\right) + 2(x) + C$$
$$= \frac{-1}{3}x^6 + \frac{3}{2}x^2 + 2x + C$$

Problem 2 $\int \sin(2s) + s ds =$

$$= \int \sin(2s)\,ds + \int s\,ds$$
$$= \frac{-1}{2}\cos(2s) + \frac{s^2}{2} + C$$

Problem 3 Solve the initial value problem:

$$\frac{dy}{dx} = 3x + 2, \quad y(0) = -1$$

$\Rightarrow \; y(x) = \int 3x + 2 \, dx$

$= 3\int x \, dx + 2\int 1 \, dx$

$= 3\left(\frac{x^2}{2}\right) + 2(x) + C$

$y(0) = 3\left(\frac{0^2}{2}\right) + 2(0) + C = C \Rightarrow C = -1$

$\Rightarrow \; y(x) = \frac{3}{2}x^2 + 2x - 1$

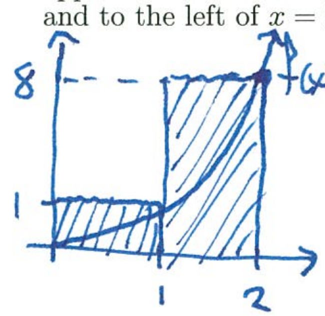
Problem 4 Find the generalized antiderivative of

$$g(x) = \frac{1}{1 + (3x)^2}$$

$\int g(x) dx = \int \frac{1}{1+(3x)^2} dx = \frac{1}{3} \arctan(3x) + C$

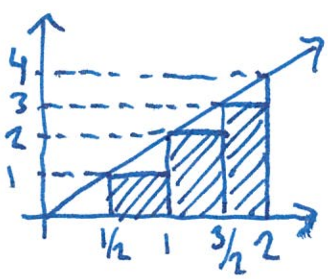
Problem 5 $\int 2^x dx =$ $\frac{2^x}{\ln(2)} + C$

Problem 6 Using an upper sum approximation with two rectangles of equal width, approximate the area of the region that lies below the graph of $f(x) = x^3$, above the x-axis, and to the left of $x = 2$.



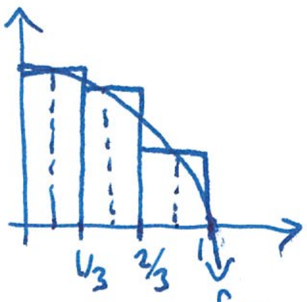
$$(\text{Area of rectangles}) = (1)(1) + (8)(1)$$
$$= 1 + 8 = 9$$

Problem 7 Using a lower sum approximation with four rectangles of equal width, approximate the area of the region that lies below the graph of $f(x) = 2x$, above the x-axis, and to the left of $x = 2$.



$$(\text{Area of rectangles}) = (0)(\tfrac{1}{2}) + (1)(\tfrac{1}{2}) + (2)(\tfrac{1}{2}) + (3)(\tfrac{1}{2})$$
$$= \frac{1}{2}(0 + 1 + 2 + 3)$$
$$= \frac{6}{2} = 3$$

Problem 8 Using a sum approximation with the midpoint method with three rectangles of equal width, approximate the area of the region that lies below the graph of $f(x) = 1 - x^2$, above the x-axis, and to the right of $x = 0$.



$$(\text{Area of rectangles}) = \left(1 - \left(\frac{1}{6}\right)^2\right)\left(\frac{1}{3}\right) + \left(1 - \left(\frac{3}{6}\right)^2\right)\left(\frac{1}{3}\right) + \left(1 - \left(\frac{5}{6}\right)^2\right)\left(\frac{1}{3}\right)$$

$$= \left(\frac{1}{3}\right)\left(\left(\frac{36}{36} - \frac{1}{36}\right) + \left(\frac{36}{36} - \frac{9}{36}\right) + \left(\frac{36}{36} - \frac{25}{36}\right)\right)$$

$$= \left(\frac{1}{3}\right)\left(\frac{35 + 27 + 11}{36}\right)$$

$$= \frac{72}{3(36)} = \frac{2(36)}{3(36)} = \frac{2}{3}$$