MATH 2250 Calculus I
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Name: Answers
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Practice Quiz No. 4

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 Find the ~~first three~~ derivatives of

$$f(x) = \csc(e^x)$$

Let $g(x) = \csc(x)$. Remember or derive $g'(x) = \frac{d}{dx}\left(\frac{1}{\sin(x)}\right)$

$= \frac{d}{dx}\left((\sin(x))^{-1}\right) = -(\sin(x))^{-2}(\cos(x))$ (Chain rule)

$= -\csc(x)\cot(x)$

Now, $f(x) = g(h(x))$, where $h(x) = e^x$, $h'(x) = e^x$,

$\Rightarrow f'(x) = g'(h(x)) \cdot h'(x) = -\csc(e^x)\cot(e^x)e^x$

Problem 2 Find the derivative of

$$f(x) = e^{2x^2+5x}$$

$f(x) = g(h(x))$, where $g(x) = e^x$, $g'(x) = e^x$

$h(x) = 2x^2 + 5x$, $h'(x) = 4x+5$

$\Rightarrow f'(x) = \frac{d}{dx}(g(h(x))) = g'(h(x)) \cdot h'(x)$

$= \left(e^{2x^2+5x}\right) \cdot (4x+5)$

Problem ~~4~~ 3 Find the derivative of

$$g(y) = \sec(\tan(y))$$

$g(y) = f(h(y))$, where $f(y) = \sec(y)$
$h(y) = \tan(y)$

Remember or derive: $f'(y) = \frac{\partial}{\partial y}\left(\frac{1}{\cos(y)}\right)$

$$= \frac{\partial}{\partial y}\left((\cos(y))^{-1}\right)$$

$$= -(\cos(y))^{-2}(-\sin(y))$$

$$= \sec(y)\tan(y)$$

and $h'(y) = \frac{\partial}{\partial y}(\tan(y)) = \frac{\partial}{\partial y}\left(\frac{\sin(y)}{\cos(y)}\right)$

$$= \frac{\cos(y)\left(\frac{\partial}{\partial y}\sin(y)\right) - \sin(y)\left(\frac{\partial}{\partial y}\cos(y)\right)}{\cos^2(y)}$$

$$= \frac{\cos^2(y) + \sin^2(y)}{\cos^2(y)} = \frac{1}{\cos^2(y)} = \sec^2(y)$$

$\Rightarrow g'(y) = f'(h(y)) \cdot h'(y) = \sec(\tan(y))\tan(\tan(y))\sec^2(y)$

Problem ~~5~~ 4 Find the derivative of

$$f(x) = e^{e^x}$$

$f(x) = g(h(x))$, where $g(x) = e^x$, $g'(x) = e^x$

$h(x) = e^x$, $h'(x) = e^x$

$\rightarrow \quad f'(x) = g'(h(x)) \cdot h'(x)$

$= e^{(e^x)} \; e^x$

$= e^{(x+e^x)}$

Problem ~~6~~ 5 Given the equation $e^{2x} = \sin(x + 3y)$, find $y'(x)$.

$$e^{2x} = \sin(x + 3(y(x)))$$

$$\frac{\partial}{\partial x}\left(e^{2x}\right) = \frac{\partial}{\partial x}\left(\sin(x + 3(y(x)))\right)$$

$$e^{2x} \cdot 2 = \cos(x + 3y(x)) \cdot (1 + 3y'(x))$$

$$1 + 3y'(x) = \frac{2e^{2x}}{\cos(x + 3y(x))}$$

$$y'(x) = \frac{1}{3}\left(\frac{2e^{2x}}{\cos(x + 3y(x))} - 1\right)$$

Problem ~~7~~ 6 Given the equation $\theta^{1/2} + r^{1/2} = 1$, find $r'(\theta)$.

$$\theta^{1/2} + (r(\theta))^{1/2} = 1$$

$$\frac{d}{d\theta}\left(\theta^{1/2}\right) + \frac{d}{d\theta}\left((r(\theta))^{1/2}\right) = \frac{d}{d\theta}(1)$$

$$\frac{1}{2}\theta^{-1/2} + \frac{1}{2}(r(\theta))^{-1/2} r'(\theta) = 0$$

$$\frac{1}{2}(r(\theta))^{-1/2} r'(\theta) = \frac{-1}{2}\theta^{-1/2}$$

$$r'(\theta) = \frac{\frac{-1}{2}\theta^{-1/2}}{\frac{1}{2}(r(\theta))^{-1/2}}$$

$$r'(\theta) = -(r(\theta))^{1/2}\theta^{-1/2}$$

Problem ~~8~~ 7 Given the equation $\cos(r) + \cot(\theta) = e^{r\theta}$, find $r'(\theta)$.

$$\cos(r(\theta)) + \cot(\theta) = e^{(r(\theta))\theta}$$

$$\frac{d}{d\theta}\left(\cos(r(\theta)) + \cot(\theta)\right) = \frac{d}{d\theta}\left(e^{\theta \cdot r(\theta)}\right)$$

$$-\sin(r(\theta))\, r'(\theta) + (-\csc^2(\theta)) = \frac{d}{d\theta}\left(f(g(\theta))\right)$$

where $f(\theta) = e^{\theta}$ and $g(\theta) = \theta \cdot r(\theta)$

$f'(\theta) = e^{\theta}$, $g'(\theta) = (1)r(\theta) + \theta r'(\theta)$

$\Rightarrow$

$$-\sin(r(\theta))\, r'(\theta) - \csc^2(\theta) = \left(e^{(\theta r(\theta))}\right)\left(r(\theta) + \theta r'(\theta)\right)$$

$$-\sin(r(\theta))\cancel{r'(\theta)} \left(e^{\theta r(\theta)}\right)\theta\, r'(\theta) = \left(e^{\theta r(\theta)}\right) r(\theta) + \csc^2(\theta)$$

$$(r'(\theta))\left(-\sin(r(\theta)) - \left(e^{\theta r(\theta)}\theta\right)\right) = r(\theta)e^{\theta r(\theta)} + \csc^2(\theta)$$

$$r'(\theta) = \frac{r(\theta)e^{\theta r(\theta)} + \csc^2(\theta)}{-\sin(r(\theta)) - \theta e^{\theta r(\theta)}}$$