MATH 1113 Precalculus
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Name: Solutions
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Practice Test No. 3

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 Find an angle coterminal to θ on the interval $[0, 2\pi]$

a $\frac{-35\pi}{4}$

$$-\frac{35\pi}{4} + 5(2\pi) = \boxed{\frac{5\pi}{4}}$$

b $\frac{19\pi}{8}$

$$\frac{19\pi}{8} - 2\pi = \boxed{\frac{3\pi}{8}}$$

Problem 2 Convert the given angles

a $\frac{2\pi}{3}$ radians to degrees.

$$\frac{2\pi}{3}\left(\frac{180^\circ}{\pi}\right) = \frac{360^\circ}{3} = 120^\circ$$

b 36° to radians

$$36^\circ = 36\left(\frac{\pi}{180}\right) = \frac{\pi}{5}$$

c $\frac{3\pi}{2}$ to revolutions (yes, revolutions, not radians)

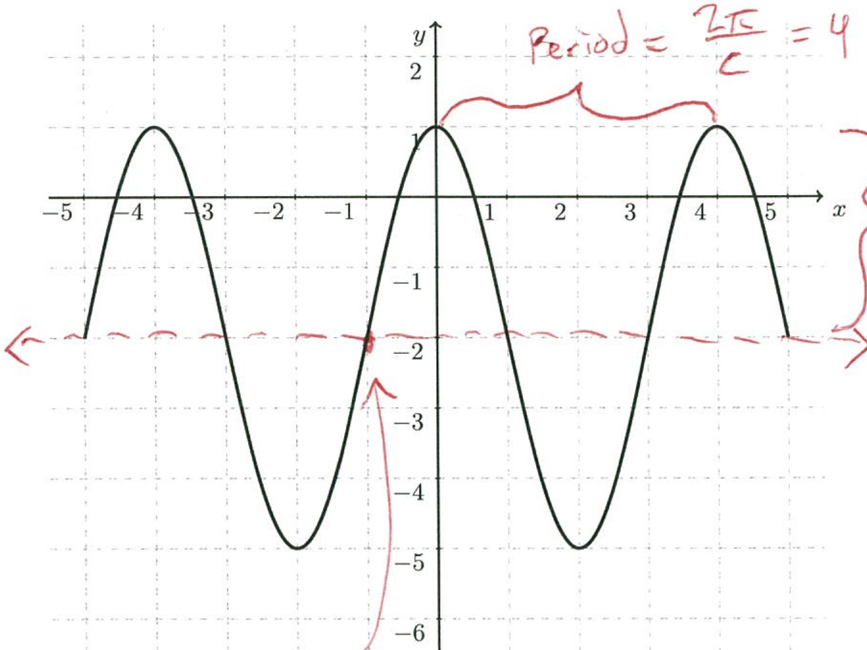
$$\frac{3\pi}{2}\left(\frac{1 \text{ rev}}{2\pi}\right) = \frac{3}{4} \text{ rev}$$

Problem 2 The function below is defined by

$$f(x) = A\sin(cx + d) + b.$$

Determine the values of the constants A, b, c, and d where A is a positive number (There is more than one correct answer).

Period $= \frac{2\pi}{c} = 4 \Rightarrow 4c = 2\pi \Rightarrow c = \frac{\pi}{2}$



$A = 3$

$y = -2 = b$

$(-1,-2) \Rightarrow f(-1) = -2$

$\Rightarrow A\sin(c(-1) + d) + b = -2$

$3\sin\left(\frac{-\pi}{2} + d\right) - 2 = -2$

$\sin\left(d - \frac{\pi}{2}\right) = 0$

Remember $\sin(0) = 0$, so

$d - \frac{\pi}{2} = 0$ is one solution.

$\Rightarrow d = \frac{\pi}{2}$

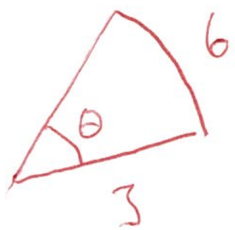
$A = 3$

$b = -2$

$c = \frac{\pi}{2}$

$d = \frac{\pi}{2}$

Problem 3 You are given that the length of an arc on a circle is 6, and that the radius of the circle is 3. What angle (in radians, of course) centered at the center of the circle gives you this arc?



$$S = 3\theta = 6$$

$$\Rightarrow \theta = \frac{6}{3} = 2$$

Problem 4 Given that the area of a circular sector is 8π and that the angle at the center of the circle that gives you this sector is $\pi/6$, what is the radius of the circle?

$$A = \left(\frac{\theta}{2\pi}\right)\pi r^2 = \frac{1}{2}\theta r^2$$

$$\Rightarrow 8\pi = \frac{1}{2}\left(\frac{\pi}{6}\right) r^2$$

$$\Rightarrow 96 = r^2$$

$r = \sqrt{96}$, because $r > 0$.

Problem 5 Graph the functions:

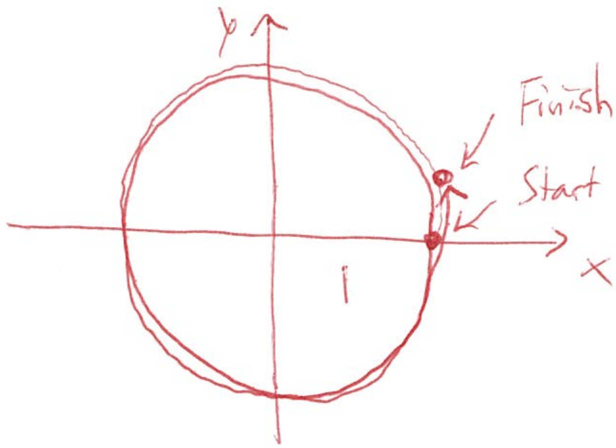
a $2\cos(x + \pi/3)$

Check w/ graphing calculator.

b $3\sin(2x + \pi/3)$

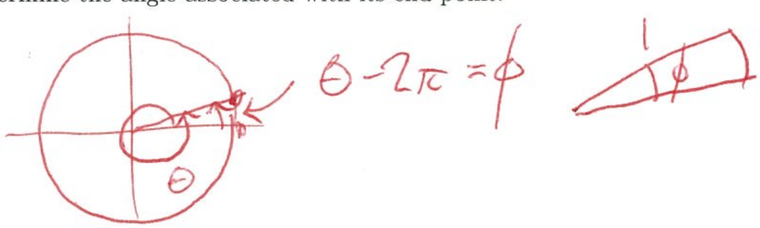
Problem 6 An ant starts at the point $P(1,0)$, and it moves counter-clockwise around a circle of radius one meter that is centered at the origin. It moves a distance of 6.5 meters.

a Make rough sketch of its position and the path it traveled. Label its start and end point.



$6.5 > 2(\pi)$, So the ant mov at least 1 full circuit of the circle.

b Determine the angle associated with its end point.



$\theta - 2\pi = \phi$

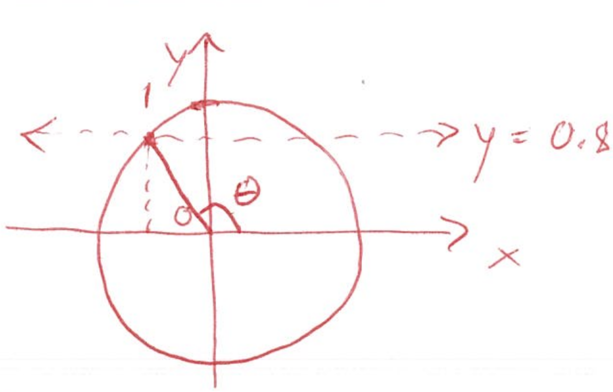
$6.5 - 2\pi$

$s = r\phi$

$6.5 - 2\pi = (1)\phi = \phi$

$\Rightarrow \boxed{\theta = 6.5}$

c The y coordinate of a point on the unit circle is 0.8, and the point is in the second quadrant. Determine the sine, cosine, and tangent of the associated angle.



$\sin(\theta) = 0.8$

$\cos^2(\theta) + \sin^2(\theta) = 1$

$\cos^2(\theta) + (.8)^2 = 1$

$\sqrt{\cos^2(\theta)} = \sqrt{1-(.8)^2}$

$\cos(\theta) = -\sqrt{1-(.8)^2}$

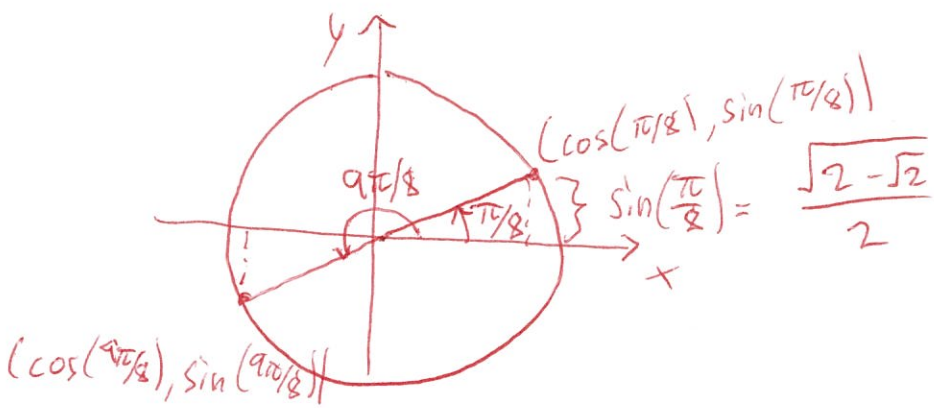
Because $\cos(\theta)$ is < 0.

$\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)} = \dfrac{0.8}{-\sqrt{1-(.8)^2}}$

Problem 7 You are browsing the web for interesting trigonometric identities, and one of your favorite pages indicates that

$$\sin\left(\frac{\pi}{8}\right) = \frac{\sqrt{2-\sqrt{2}}}{2}.$$

Use this information to determine the exact value of $\sin\left(\frac{9\pi}{8}\right)$. (Explain and provide a full justification for your answer. The number by itself is not worth any credit.)

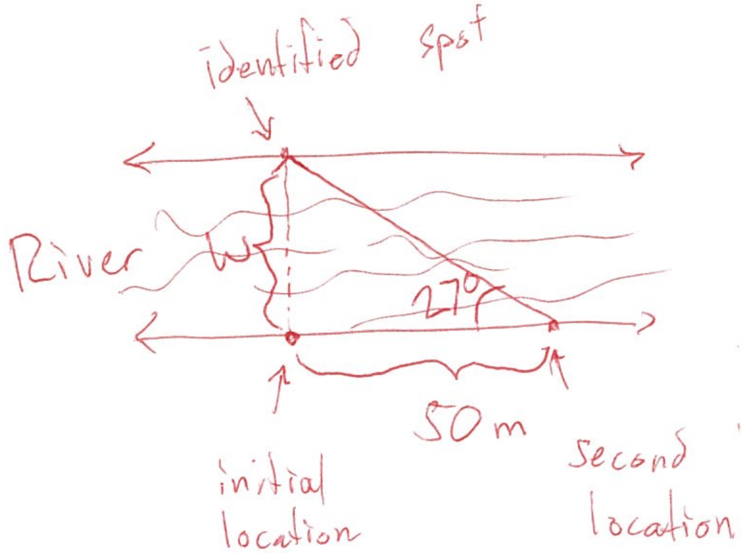


Because $(\cos(\pi/8), \sin(\pi/8))$ and $(\cos(9\pi/8), \sin(9\pi/8))$ are symmetric about the origin, $\cos(\pi/8) = -\cos(9\pi/8)$ and

$\sin(\pi/8) = -\sin(9\pi/8)$

$\Rightarrow \sin(9\pi/8) = -\sin(\pi/8)$

Problem 8 A surveyor is asked to determine the width of a river. The surveyor identifies a spot directly across the river and then walks 50 meters downstream. The surveyor determines that the angle formed between the line along the river and the current line of sight to the original spot on the other side is 27°. How wide is the river?



$$\tan(27°) = \frac{w}{50}$$

$$\Rightarrow \quad 50\tan(27°) = w$$

$$50\tan\left(27\left(\frac{\pi}{180}\right)\right) = w$$

If you want radians

So the river is $50\tan(27°)$ meters wide.